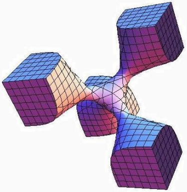
Counting points on the representation variety

Bailey Whitbread

AustMS 2022, December 6-9.



A representation space [CFLO].

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non-abelian Hodge theory: $\mathrm{Higgs}_G(X)/\!/\mathcal{G} \leftrightarrow \mathrm{Hom}(\pi_1(X), G)/\!/G$

Riemann-Hilbert correspondence: $\mathrm{Hom}(\pi_1(X), G)/\!/G \leftrightarrow \mathrm{LocSys}_G(X)$

$\underbrace{\mathrm{Higgs}_G(X)/\!/\mathcal{G}}_{\text{gauge theory, integrable systems, mirror symmetry}}$

$\underbrace{\mathrm{LocSys}_G(X)}_{\text{sheaf cohomology, flat connections, } D\text{-modules}}$

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$$\Gamma := \frac{\langle x_1, y_1, \ldots, x_g, y_g, z \rangle}{[x_1, y_1] \ldots [x_g, y_g] z} = \pi_1 \Bigg($$

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The representation variety $\mathbf{R}(G, \Gamma, C)$ associated to this data is

$$\mathbf{R} := \left\{ (x_1, y_1, \ldots, x_g, y_g, z) \in G^{2g} \times C \;\middle|\; [x_1, y_1] \ldots [x_g, y_g] z = 1 \right\}.$$

E-polynomials and their properties

We want to understand the topology of the representation variety. In particular, we seek an expression for the *E-polynomial* of $\mathbf{R}$, denoted $E(\mathbf{R}; x, y) \in \mathbb{Z}[x, y]$.

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For a complex variety $\mathbf{X}$, the E-polynomial $E(\mathbf{X}; x, y)$ carries an abundunce of topological information:

(i) The dimension of $\mathbf{X}$ is half of the degree of $E(\mathbf{X}; x, y)$,

(ii) The Euler characteristic of $\mathbf{X}$ is $E(\mathbf{X}; 1, 1)$,

(iii) The $\#$ of (max'l dimension) irred. components of $\mathbf{X}$ is the leading coefficient of $E(\mathbf{X}; x, y)$.

Katz' theorem

Theorem [Katz]

Let $\mathbf{X}$ be a variety. Assume that $|\mathbf{X}(\mathbb{F}_q)| = P_{\mathbf{X}}(q)$ for some polynomial $P_{\mathbf{X}} \in \mathbb{Z}[q]$.

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For example,

$$|\mathrm{GL}_2(\mathbb{F}_q)| = q^4 - q^3 - q^2 + q = P_{\mathrm{GL}_2}(q)$$

$$\text{dimension} = 4, \ \text{Euler characteristic} = 0,$$

$$\text{no. of irred. components} = 1.$$

The Frobenius mass formula

Theorem [Frobenius 1896, Mednykh 1978]

$$|\mathbf{R}(\mathbb{F}_q)| = |C(\mathbb{F}_q)| \sum_{\chi \in \mathrm{Irr}(G(\mathbb{F}_q))} \left(\frac{|G(\mathbb{F}_q)|}{\chi(1)} \right)^{2g-1} \chi(s).$$

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Understand $\mathrm{Irr}(G(\mathbb{F}_q))$ $\xrightarrow{\text{mass formula}}$ Obtain $|\mathbf{R}(\mathbb{F}_q)|$ and $E(\mathbf{R}; q)$

This turns the problem of algebraic geometry into a problem of representation theory.

Recollections of representation theory

Theorems of Deligne-Lusztig, Curtis-Iwahori-Kilmoyer and Tits tell us that we need to look at:

- Stabiliser subgroups W_θ, where $W \curvearrowright \theta \in \mathrm{Irr}(T(\mathbb{F}_q))$, and
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$$T(\mathbb{F}_q) = \begin{pmatrix} \mathbb{F}_q^\times & & \\ & \ddots & \\ & & \mathbb{F}_q^\times \end{pmatrix}$$

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whose characters look like

$$\theta_{\alpha_1,\ldots,\alpha_n} \begin{pmatrix} t_1 & & \\ & \ddots & \\ & & t_n \end{pmatrix} := \alpha_1(t_1)\cdots\alpha_n(t_n), \quad \alpha_i \in \mathrm{Irr}(\mathbb{F}_q^\times).$$

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The Weyl group $W \simeq S_n$ acts via permutating the α_i's:

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So their stabilisers look like

$$W_{\theta_{\alpha_1,\dots,\alpha_n}} \simeq S_{n_1} \times \cdots \times S_{n_r}, \quad \text{where } n_1 + \cdots + n_r = n.$$

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For GL_3, this is iso. to the lattice of set-partitions of $\{1, 2, 3\}$:

Generic degrees

Each $\lambda \in \mathrm{Irr}(W_\theta)$ has an associated generic degree $D_\lambda \in \mathbb{Q}[q]$.

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The coefficients tell you how $\mathcal{B}(\theta)$ decomposes!

$$\mathcal{B}(\theta) = V_1^{\oplus 1} \oplus V_2^{\oplus 2} \oplus V_3^{\oplus 1}.$$

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Theorem [Hausel, Letellier, Rodriguez-Villegas, '11]

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$$E(\mathbf{R}; q) = q^{\frac{1}{2} d_C} \frac{|\mathrm{GL}_n(\mathbb{F}_q)|}{|Z(\mathrm{GL}_n(\mathbb{F}_q))|} \mathbb{H}_C(q^{1/2}, q^{-1/2}).$$

Going forward

$E(\mathbf{R}; q)$

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↑ G=reductive

Our formula

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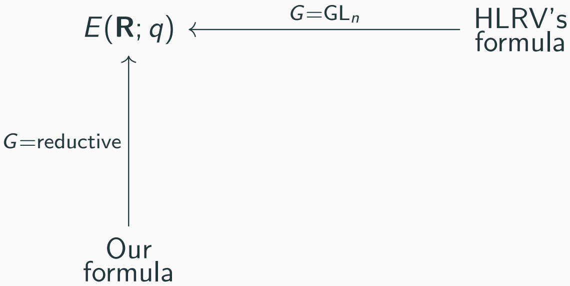

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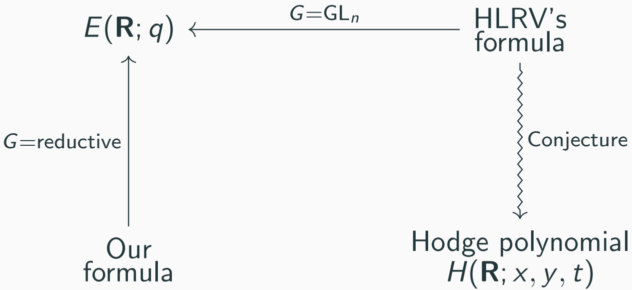

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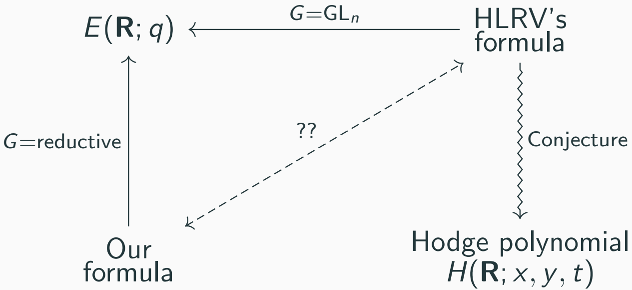

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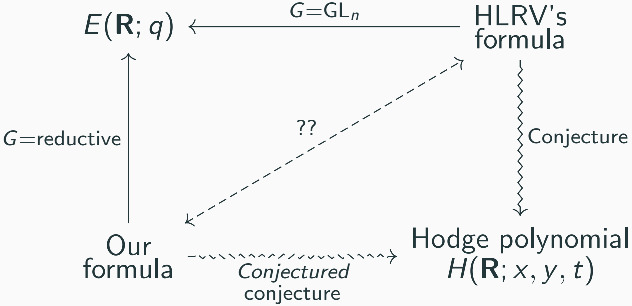

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